Theory of Intracavity Frequency Doubling in Passively Mode-Locked Femtosecond Lasers

TIAN RONG ZHANG, GLENN FOCHT, PAUL E. WILLIAMS, AND M. C. DOWNER

Abstract—**We construct a quantitative theoretical model of an intracavity frequency doubled and passively mode-locked laser, supported by experimental observations with a colliding pulse mode-locked femtosecond dye laser. The major findings are that for second harmonic conversion efficiencies consistent with continuing laser operation (<5 percent), 1) a stable mode-locking regime always exists, although it narrows somewhat with increasing conversion efficiency; 2) the duration of the fundamental pulses can always be preserved, even in the femtosecond time domain, by readjusting saturable gain and saturable loss parameters; 3) the energy of the fundamental pulses can also be preserved under the same conditions. Both the model and observations contrast with previous studies of actively mode-locked and synchronously mode-locked lasers containing intracavity frequency doubling crystals.**

I. Introduction

We RECENTLY demonstrated experimentally [1], [2] that intracavity frequency doubling of a visible wavelength passively mode-locked femtosecond dye laser could produce perfectly synchronized femtosecond ultraviolet and visible pulse trains, each at 100 MHz repetition rate and milliwatt average power. More significantly, we achieved this result while preserving the pulse duration, frequency bandwidth, power, and mode-locking stability of the fundamental laser output. This result contrasted sharply with previous studies which showed significant pulse broadening, bandwidth limitation, and mode-locking instability caused by an intracavity frequency doubler in actively mode-locked solid-state lasers [3] and synchronously mode-locked dye lasers [4]–[6], even though these lasers operated in the picosecond, rather than the femtosecond, time domain. In this paper, we examine quantitatively the underlying physical mechanisms responsible for the more favorable performance of intracavity frequency doubled *passively* mode-locked lasers. Previous theoretical analyses have examined the effect of intracavity frequency doubling on the actively [3] and synchronously [5] mode-locked lasers. Our analysis, however, is the first to examine the effect of intracavity frequency doubling on the operation of *passively* mode-locked lasers.

Manuscript received December 30, 1987; revised April 6, 1988. This work was supported by the Joint Services Electronics Program under Contract F49620-86-C0045, the Robert A. Welch Foundation under Grant F-1038, and the National Science Foundation under Grant DMR-8858388. The work of M. C. Downer was also supported by an IBM Faculty Development Award.

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IEEE Log Number 8822027.

We construct a model, based on Haus's theory of the passively mode-locked laser [7], which shows that saturable loss and gain media, absent in the synchronously mode-locked [4]–[6] and actively mode-locked [3] lasers, can compensate the destabilizing and pulse broadening influence of an intracavity frequency doubling crystal. Furthermore, our model defines quantitatively the regimes of saturable loss, saturable gain, and second harmonic conversion efficiency in which stable mode-locked operation is possible, as well as the effect of the frequency doubler on pulse duration and pulse energy within the stable regime. Finally, we compare these theoretical results with observations using an intracavity KDP doubling crystal. While we experimentally demonstrated intracavity frequency doubling in a colliding pulse mode-locked laser [8], our model assumes no features unique to that laser. Our results suggest, therefore, that stable intracavity frequency doubled operation, with synchronized ultraviolet and visible outputs of comparable power and pulse duration, should be generally achievable in all passively mode-locked lasers which employ saturable gain and loss media.

II. Fundamental Mechanisms

Fig. 1 depicts the essential mechanisms by which saturable loss and gain media having relaxation times longer than the pulse duration can compensate the destabilizing and pulse broadening effect of an intracavity frequency doubler. The intracavity fundamental pulse suffers temporal broadening in the doubling crystal because power dependent loss selectively attenuates the peak of the pulse. Subsequent passes multiply this pulse broadening effect, as noted in previous analyses [3], [5]. Passage through a saturable absorber, on the other hand, selectively attenuates and therefore sharpens the leading edge of the pulse. Analogously, saturation of the gain then sharpens the trailing edge. Appropriate adjustment of absorber and gain saturation levels can therefore precisely compensate the pulse broadening caused by the doubling crystal. In fact, a major result of our analysis is that such compensation is always possible in a passively mode-locked laser. Such compensating mechanisms were absent in the actively mode-locked [3] and synchronously mode-locked [4]–[6] lasers.

The doubling crystal introduces other sources of pulse broadening besides that shown in Fig. 1. For example, linear group velocity dispersion broadens the fundamental

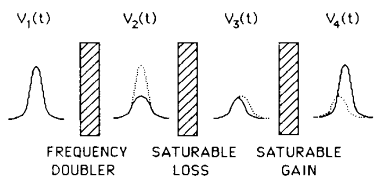

Fig. 1. Changes in the intracavity pulse envelope upon passage through major components of a passively mode-locked laser with an intracavity frequency doubler.

pulses, particularly those in the femtosecond domain. Such dispersive broadening, however, is routinely compensated by a negatively dispersive intracavity prism configuration [9]. In addition, the second harmonic pulses are broadened because of group velocity walk-off of the fundamental and second harmonic pulses, necessitating a thin doubling crystal to generate short ultraviolet pulses. However, the duration of the fundamental pulse, our chief concern, is unaffected by this walk-off. These additional broadening mechanisms will therefore not be considered further in our analysis.

III. Theoretical Framework

In order to analyze the events in Fig. 1 quantitatively, we augment Haus's model [7] of a passively mode-locked laser with a frequency doubling element. While Haus analyzed both the cases of saturable absorbers with relaxation times which were slow [7] and fast [10] compared to the pulse duration, we confine our analysis to the case of the slow saturable absorber, which has resulted in the shortest pulse laser sources, since the effects of the intracavity frequency doubler are greatest in these shortest pulse lasers. We retain Haus's basic assumption that the intracavity pulse is perturbed only modestly upon passing through each intracavity element, thus allowing perturbation expansions to be truncated at low orders. This simplifying assumption leads to an analytic solution for the pulse envelope function, and permits quantitative insight into basic physical mechanisms, although it inevitably limits the accuracy of our description of actual laser cavities to some extent, as discussed more fully below. We also omit explicit treatment of the effects of self-phase modulation and group velocity dispersion because, as noted above, addition of the frequency doubler introduces no substantive new elements into previous analyses of these effects [11]. Nevertheless, our analysis could easily be extended along the lines of Martinez *et al.* [11].

As in Haus's analysis [7], we trace the propagation of the normalized pulse electric field envelope $V(t)$ around the optical cavity shown in Fig. 1. The envelope function $V(t)$ is defined such that the cumulative energy of the pulse is

$$E(t) = \int_{-\infty}^{t} |V(t')|^2 \, dt'. \tag{1}$$

In addition to the elements depicted, we also include a bandwidth-limiting element and linear loss, for completeness. We require that the pulse envelope repeat itself after a round-trip traversal of the cavity. The depleted fundamental pulse envelope $V_2(t)$ after the doubling crystal can be related to the pulse envelope $V_1(t)$ incident on the crystal by the expression [3], [12]

$$V_2(t) = V_1(t) \operatorname{sech}\left(KV_1(t)\right) \tag{2}$$

where K is a constant related to the second harmonic conversion efficiency. In order for the laser to continue operating, the conversion efficiency must be small, i.e., $KV_1(t) << 1$. To lowest order, the second harmonic power decreases the fundamental power, i.e.,

$$|V_2(t)|^2 = |V_1(t)|^2 - K^2 |V_1(t)|^4. \tag{3}$$

Then taking the square root of the above expression, the output of the doubling crystal is then approximated by

$$\begin{aligned} V_2(t) &\approx V_1(t)\left(1 - \frac{K^2}{2}|V_1(t)|^2\right) \\ &\approx V_1(t) \exp\left(-\frac{K^2}{2}|V_1(t)|^2\right). \end{aligned} \tag{4}$$

We will find it convenient to describe second harmonic conversion efficiency in terms of the dimensionless parameter

$$\gamma = \frac{K^2 E_A \omega_c}{2\alpha} = \frac{3}{2} \frac{E_A \beta \omega_c \tau_p}{E\alpha} \tag{5}$$

where β is the average second harmonic conversion efficiency, E_A is the saturation energy for the absorber, $E = E(\infty)$ is the total energy of the pulse, τ_p is the pulse duration, ω_c is the width of the loss "well" of the bandwidth-limiting element, and $\alpha = \omega_o T_R / Q$ is the linear cavity loss, including the linear insertion loss of the doubling crystal, with ω_o the center frequency of the mode spectrum, T_R the free-space cavity round-trip time, and Q denoting the "Q" of the optical cavity. As a numerical example, the value $\gamma = 0.15$ indicates a conversion efficiency of 0.3 percent for 70 fs 5 nJ intracavity pulses passing through a 1 mm KDP crystal at the type I phase matching angle [2]. Our assumption of small conversion efficiencies is equivalent to restraining γ to values of approximately $\gamma < 2$.

Our description of the remaining elements in the cavity closely follows that of Haus [7]. The modification of the pulse envelope in the saturable gain and saturable absorber is described by the transfer functions $\exp(A(t))$ and $\exp(-B(t))$, respectively, where $A(t)$ and $B(t)$ are the gain coefficient and loss coefficient. Hence we can write $V_3(t) = V_2(t) \exp(-B(t))$ and $V_4(t) = V_3(t) \exp(A(t))$. An analogous expression relates the pulse envelope $V_5(t)$ after the bandwidth-limiting element to $V_4(t)$, where the transfer function is obtained by expanding the spectrum transfer function [7] to second order in powers of $(\omega - \omega_0)$ and replacing $i(\omega - \omega_0)$ by d/dt. The general equation for the pulse envelope is then obtained by requiring repetition of the pulse envelope after one round-trip:

$$\begin{aligned} V_5(t) &= \left[\exp\left(-K^2V_1^2(t)/2 - B(t) + A(t)\right.\right. \\ &\quad \left.\left. - \alpha(1 - \omega_c^{-2}\, dt/dt^2)\right)\right] V_1(t) \\ &= V_1(t - T_R + \delta T) \end{aligned} \tag{6}$$

where δT is a time-delay parameter indicating deviation of the pulse repetition period from the free-space cavity round-trip time T_R. Using the assumption of small change of the pulse in one round-trip, the total exponential transfer function can be expanded to first order. Expanding the final envelope function of the pulse to first order in delay time δT, we obtain a differential equation for the pulse envelope:

$$-\frac{1}{\omega_c^2}\frac{d^2V}{dt^2} + \frac{2\delta T}{\alpha}\frac{dV}{dt} - g(U)\,V(t) = 0 \tag{7}$$

where

$$\begin{aligned} g(U) = &-(1 + q - g^{(i)}) + \left[\left(q - \frac{g^{(i)}}{s}\right) - \frac{qV_0}{2}\right. \\ &\left.\cdot\left(1 - \frac{1}{\mu^2}\right)\right] U - \frac{q}{2\mu^2}U^2 \end{aligned} \tag{8}$$

is the net gain function for the cavity with doubling crystal, with $U = E(t)/E_A$ as the normalized energy and the constants μ, V_0 defined by

$$\mu = \frac{\gamma}{\sqrt{q}} + \sqrt{1 + \frac{\gamma^2}{q}} \quad \text{and} \quad V_0 = \frac{E}{E_A}. \tag{9}$$

Equations (7) and (8) correspond to (12) and (34) of Haus [7], with the addition that the doubling crystal contributes an intensity dependent negative gain $-(2\gamma/E_A\omega_c)\,V^2(t)$ as anticipated in Fig. 1. The following symbols used in (7) and (8) were introduced by Haus in [7]: $g^{(i)}$ stands for the gain before arrival of the pulse; q is the measure of small-signal loading of the saturable absorber; New's parameter [13] $s = E_L/E_A$ is the ratio of saturation energies for gain medium and for saturable absorber.

A solution of (7) is

$$V(t) = \frac{\sqrt{V_0E_A/2\tau_p}}{\cosh(t/\tau_p)} \tag{10}$$

where V_0 and the pulse duration τ_p are determined by substitution of (10) into (7). In particular

$$V_0 = \frac{1}{\left(1 - \frac{1}{4\mu^2}\right)}\left[\left(1 - \frac{g^{(i)}}{qs}\right) + \sqrt{\left(1 - \frac{g^{(i)}}{qs}\right)^2 - \frac{4}{q}\left(1 - \frac{1}{4\mu^2}\right)(1 + q - g^{(i)})}\right] \tag{11}$$

and τ_p is related to V_0 by

$$\tau_p V_0 = \frac{4\mu}{\sqrt{q}\,\omega_c}. \tag{12}$$

Numerical values of the various cavity parameters can be estimated for an actual laser such as the colliding pulse laser. We estimate basic cavity parameters as follows: linear loss $\alpha = 0.04$, center frequency $\omega_0 = 3 \times 10^{15}\ \mathrm{s}^{-1}$ ($\lambda_0 = 620$ nm), round-trip time $T_R = 10$ ns, $\omega_c = 2.7 \times 10^{13}\ \mathrm{s}^{-1}$, saturation energy of the rhodamine 6*G* gain medium E_L is about 100 nJ [14].

For the commonly used saturable absorber DODCI, estimation of the saturation energy is complicated by simultaneous presence of a normal species and a photoisomer [14], which have different saturation energies at the normal operating wavelength (620 nm). As a further complication, the saturation energy for the photoisomer (0.4 nJ) is smaller than typical intracavity pulse energies [14], thus invalidating the small saturation approximation $E < E_A$ required for the analytic treatment used here. Effective saturation energies for both absorber species can also be reduced by the colliding pulse effect [15], [16]. In order to circumvent these difficulties, we can introduce a single "effective" saturation energy $E_A \approx 3.5$ nJ, which approximates the combined action of the normal species and photoisomer without violating the approximation of small saturation too seriously. Using these physically reasonable numbers, we calculate pulse energy $E = 7$ nJ, and pulse duration $\tau_p = 70$ fs at $q = 1.2$, $g^{(i)} = 1.95$ without the doubling crystal ($\gamma = 0$). Using (11) and (12), at $q = 1.53$, $g^{(i)} = 2.276$, and a 1 mm thick intracavity KDP crystal ($\gamma = 0.15$), we arrive at the same pulse energy and pulse duration. These results agree with observed values for the colliding pulse laser [2]. Hence our model, noting the caveats described above, satisfactorily simulates the actual operation of the colliding pulse laser.

IV. Stability

In order to discuss the stable region for the laser, we examine the net gain function (8). We can describe the laser's stable regime in terms of this net gain function by means of the following three inequalities:

$$g(0) < 0 \tag{13a}$$

$$g(V_0) < 0 \tag{13b}$$

$$q - \frac{g^{(i)}}{s} - \frac{qV_0}{2}\left(1 - \frac{1}{\mu^2}\right) > 0. \tag{13c}$$

These inequalities correspond respectively to the criteria that the net gain must be negative before and after the pulse, and positive during the pulse.

We can use the inequalities (13) to construct a graphical representation of the stable region as a function of loss (q), gain ($g^{(i)}$), and γ, as shown in Fig. 2(a). The stable regime is drawn as a volume in one octant of a three-dimensional space defined by positive values of q, $g^{(i)}$, and γ. The stable volume is enclosed on the bottom by

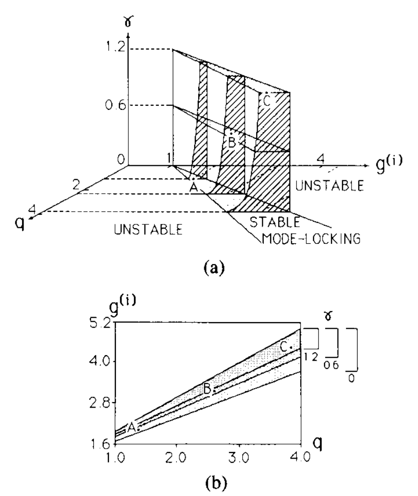

Fig. 2. (a) Graphical representation of the stable mode-locking regime as a function of saturable loss q, saturable gain $g^{(i)}$, and the dimensionless second harmonic generation parameter γ. The points A, B, and C correspond to stable operating conditions at $\gamma = 0$, 0.6, and 1.2, respectively, with equal pulse duration and pulse energy. The $(q, g^{(i)})$ coordinates of these points are (1.2, 1.95), (2.52, 3.25), and (3.84, 4.55), respectively, and are indicated by the dashed lines in the $\gamma = 0$ plane. (b) A projection of three horizontal cross sections of the stable volume corresponding to $\gamma = 0$, 0.6, 1.2, and of the three points A, B, and C, onto the $\gamma = 0$ plane. Note the narrowing of the stable regime as γ increases.

the $\gamma = 0$ plane, which corresponds to the absence of a frequency doubler, the regime of Haus's analysis [7], and on the sides by surfaces defined by the inequalities (13). The cross-hatched areas show vertical cross sections of the stable volume for three constant values of the loss parameter q (1.2, 2.52, and 3.84), while the dotted areas show analogous horizontal cross sections for three values of γ (0, 0.6, and 1.2).

A number of important observations can now be made about Fig. 2(a). First, the stable volume forms a barrier between two unconnected volumes corresponding to unstable operation. The unstable volume which corresponds to the lower values of $g^{(i)}$ includes the points for which (13b) is not satisfied. Consequently net gain is positive after passage of the pulse. This volume also includes the points for which no laser action occurs and no solution exists, due to insufficient gain or excessive loss. From this volume, the stable mode-locking regime can be reached by a decrease in loss, an increase in gain, or both. In the other unstable volume, inequality (13a) is not satisfied. Consequently gain is positive even before the pulse, leading to unstable lasing. From this volume stability is recovered by an increase in loss, a decrease in gain, or both. The important observation is that for all values of second harmonic conversion efficiency shown, stable mode-locking can always be recovered by an adjustment of gain and/or loss parameters. In practice this would be done most easily by adjusting the pump laser power or by adjusting the focus of the intracavity beam on the saturable absorber. Indeed our observations [2] showed that stable mode-locking was possible at all values of second harmonic conversion efficiency up to 0.3 percent ($\gamma = 0.15$). Fig. 2(a) predicts that stable mode-locking should also be possible at higher values of γ.

Second, as γ increases, the stability regime narrows monotonically. This narrowing occurs entirely from the smaller $g^{(i)}$, larger q side of the stable volume, as shown in Fig. 2(a). Consequently an arbitrary increase in γ from within the stable volume leads ultimately to cessation of laser action, as expected. The stable volume drawn in Fig. 2(a), if continued upward indefinitely, would retain finite width up to infinite values of γ. This unphysical behavior is an artifact of our approximation of small second harmonic conversion efficiency. A more exact calculation would show this width vanishing at larger values of γ. We can approximate this actual behavior by truncating the stable volume at $\gamma \sim 2$.

In practical terms, narrowing of the stability volume with increasing γ manifests itself in increased sensitivity of the mode-locked laser to small perturbations in gain or loss. Such perturbations can arise from drifts in pump power, air currents, mechanical vibrations, or thermo-optic effects, as discussed in [2]. We have made approximate measurements in the colliding pulse laser which compare the width of the stable regime for maximum (0.3 percent) and for very small (<0.05 percent) conversion efficiencies. These measurements were made by ramping the pump power upward from a nonlasing condition, with the intracavity crystal angle-tuned for precise phase matching, and noting the difference in pump powers at which stable mode-locked operation began, on the one hand, and switched to multiple-pulse or other unstable operation, on the other hand. This measurement was then repeated with the crystal detuned far enough from phase-matching to reduce second harmonic conversion substantially, but not far enough to affect significantly the intracavity alignment. Both measurements were repeated numerous times to improve statistics; similar measurements were also made with different cavity alignments, and with a downward ramp in the pump power. The results consistently showed no difference in the width of the stable regime between maximum and small conversion efficiencies, within an experimental uncertainty of approximately ± 30 percent.

Fig. 2(a) lends quantitative content to these observations. At 0.3 percent conversion efficiency ($\gamma = 0.15$) Fig. 2(a) shows a narrowing of only $1/10$ in the width of the stable regime, consistent with our observation of no observable narrowing. More importantly, Fig. 2(a) allows us to estimate how serious this narrowing may become with scaling of conversion efficiency up to 1 or 5 percent. The graph shows clearly, however, that although narrowing continues with further increases in conversion efficiency, the rate of narrowing decreases. For example, $\gamma = 0.6$ reduces the width of the stable regime by $1/3$, but an additional doubling of conversion efficiency to $\gamma = 1.2$ reduces the width by only an additional 25 percent, or to about $1/2$ of its width at $\gamma = 0$. Fig. 2(b), which shows projections of the stable regime onto the $\gamma = 0$

plane for $\gamma = 0.06$, and 1.2, depicts this trend more clearly. Consequently, we expect that narrowing of the stability regime should not be a serious impediment to intracavity frequency doubling up to conversion efficiencies of roughly 5 percent.

The major conclusions of this section are as follows. 1) Stable frequency doubled passively mode-locked operation can always be achieved by adjusting the gain or loss, up to conversion efficiencies of a few percent. 2) The stability regime narrows with increasing second harmonic conversion; however, this narrowing is modest, and will probably not seriously inhibit practical stable operation for conversion efficiencies up to a few percent.

V. Duration and Energy of Fundamental Pulses

We now examine the characteristics of the fundamental pulses within the stable operating regime as the second harmonic conversion efficiency is varied. In particular we ask two questions. 1) How are pulse energy and pulse duration affected by a change in γ, assuming no other cavity parameters are changed? 2) Following a change in γ, can the original pulse duration and pulse energy be recovered by adjusting other cavity parameters?

From (10)–(12), we obtain the energy per pulse and pulse duration as functions of q and $g^{(i)}$ for different parameters γ. In Fig. 3(a) the solutions of these equations are plotted as three surfaces denoting the intracavity pulse duration at three different values of γ(0, 0.6, and 1.2), each as a function of q and $g^{(i)}$. Pulsewidth surfaces with larger γ lie above the surfaces of smaller γ. Consequently an increase in second harmonic conversion efficiency at a *given* saturable gain and saturable loss lengthens the pulse duration. This conclusion agrees with our observations. More important, however, is our observation [2] that readjustment of gain and loss parameters always allowed recovery of the original pulse duration. Fig. 3(a) also substantiates this observation. For example, some areas of the $\gamma = 0.6$ surface at large values of q and $g^{(i)}$ lie at the same level as, or below areas of the $\gamma = 0$ surface at small values of q and $g^{(i)}$. A similar comment applies to the $\gamma = 1.2$ surface. The points A, B, and C provide specific examples of conditions on the $\gamma = 0$, 0.6, and 1.2 surfaces, respectively, which yield the same calculated pulse duration of 70 fs. These points are also labeled in Fig. 2(a), which shows that they are inside the stable region. In general, an increase in both q and $g^{(i)}$ is required to maintain pulse duration as γ increases. Our analysis suggests that preservation of pulse duration should be possible up to $\gamma \sim 2$. At higher values, modest increases in pulse duration are probably required to maintain stable operation.

It is illuminating to contrast our algebraic expression for pulse duration with an analogous expression derived by Yamashita *et al.* [5] for the synchronously mode-locked laser. Using (12) we express the pulsewidth τ_p in terms of the pulsewidth τ_p^0, produced in the same laser without the intracavity crystal

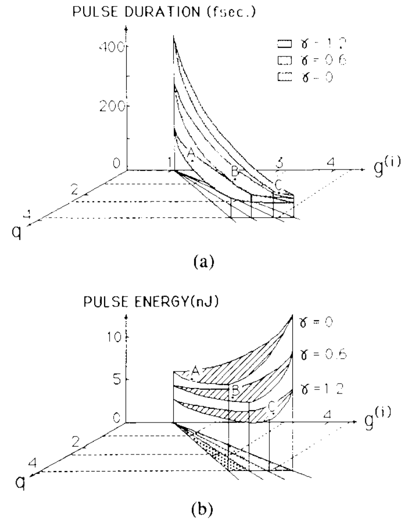

Fig. 3. Graphical representations of (a) intracavity pulse duration and (b) pulse energy at three different values of the second harmonic generation parameter γ. The points A, B, and C illustrate the simultaneous preservation of pulse duration and energy as γ is increased, and correspond to the equivalently labeled points in Fig. 2.

$$\tau_p = \mu \sqrt{\frac{q^0}{q} \frac{V_0^0}{V_0}}\, \tau_p^0. \tag{14}$$

This can be rewritten in terms of conversion efficiency γ. For small β, the pulsewidth is approximated by

$$\tau_p = \left(1 + \frac{16}{\sqrt{2} V_0^2} \frac{\beta}{\alpha q}\right) \sqrt{\frac{q^0}{q} \frac{V_0^0}{V_0}}\, \tau_p^0. \tag{15a}$$

Note that an increase of the saturable loss q and of saturable gain (manifested as an increase in V_0) compensates the pulse broadening influence of an increase in second harmonic conversion efficiency β, i.e., the pulsewidth can be adjusted to be approximately equal to the original pulsewidth: $\tau_p \sim \tau_p^0$. By contrast, Yamashita *et al.* derived an expression [5, Eq. 1] for the pulsewidth in a synchronously mode-locked dye laser, which for small β and α becomes

$$\tau_p \sim \left(1 + \sqrt{2} \frac{\beta}{\alpha}\right) \frac{V_0^0}{V_0} \tau_p^0. \tag{15b}$$

(In [5] the authors use the notation α_{SHG} for β and α_0 for α.) Note that the pulsewidth simply increases linearly with second harmonic conversion efficiency, without the possibility of compensation by an increase in saturable loss. It is easy to check from (15b) that the pulsewidth is $\tau_p \sim 1.3\tau_p^0$ for $\beta \sim 0.85$ percent. Expressions (15a) and (15b) embody the fundamental contrast between the effect of intracavity frequency doubling on the operation of a passively mode-locked laser, on the one hand, and of a synchronously mode-locked laser, on the other hand.

In Fig. 3(b) we plot analogous surfaces denoting intracavity pulse energy for $\gamma = 0$, 0.6, and 1.2. The plot

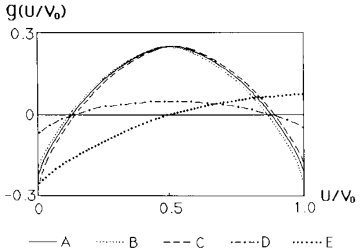

Fig. 4. Net gain parabolas showing gain versus normalized pulse energy from beginning ($U/V_0 = 0$) to end ($U/V_0 = 1$) of the pulse. Curves *A*, *B*, and *C* correspond to the points *A*, *B*, and *C* in Figs. 2 and 3, and show net loss before and after the pulse and net gain during the pulse, indicating mode-locking stability. Curve *D* depicts a stable condition of lower pulse energy and longer duration, while curve *E* depicts an unstable condition.

shows that an increase in γ at constant gain and loss parameters results in a decrease in pulse energy, as required by conservation of energy. On the other hand, the plots and observations [2] both show that the original pulse energy can be recovered by a readjustment of gain and loss parameters, as was found with pulse duration. Moreover, there is a pair of readjusted gain and loss parameters $g^{(i)}$, q such that the original pulse energy and pulse energy can be recovered simultaneously. To demonstrate this, the three points *A*, *B*, and *C* have also been plotted on Fig. 3(b), where we find that they correspond to the same pulse energy as well as to the same pulse duration.

It is also convenient to analyze the problem using the net-gain parabola, first introduced by Haus [7]. Fig. 4 compares the net-gain parabolas (*B*–*E*) with the net-gain parabola *A* of the cavity without a doubling crystal. Curves *A*, *B*, *C* correspond to second harmonic conversion efficiencies of $\beta = 0$, 1.7, and 3.4 percent, respectively, for 7 nJ pulse energy and 70 fs pulsewidth, similar to conditions in the colliding pulse laser [2]. Curve *E* depicts an unstable situation in which gain and absorber were left unchanged after insertion of the crystal. Stability can be recovered by increasing the gain only or by decreasing the saturable loss only, but at the expenses of longer pulse duration and lower pulse energy (curve *D*). By increasing both saturable gain and saturable loss, however, stable operation with the original pulse energy and duration is recovered, as depicted by *B* and *C*.

VI. Conclusions and Extensions

This paper has analyzed the characteristics of a passively mode-locked laser with an intracavity frequency doubling crystal. It has been shown that three major lasing characteristics—stable mode-locking, pulse duration, and pulse energy—can be preserved for second harmonic conversion efficiencies up to approximately 5 percent. We satisfactorily model the observed qualitative behavior, and to a large extent the quantitative characteristics, of the frequency doubled colliding pulse laser.

The remarkably stable operation and narrow pulse durations which have been observed [2] and analyzed in this work invite extensions to other intracavity nonlinear processes, such as parametric downconversion and upconversion. The former process offers the potential of infrared pulse generation in the 1 to 3 μm regime, while the latter could extend ultraviolet generation as far as 200 nm, both spectral regimes in which high repetition rate femtosecond source lasers are currently unavailable. While unseeded parametric downconversion and upconversion would be inefficient under most practically realizable intracavity conditions, synchronized injection of seed pulses from an external source could dramatically increase conversion efficiencies for these processes. For example, a $LiIO_3$ crystal could serve to downconvert intracavity femtosecond visible pulses by mixing with externally injected longer pulses from a semiconductor diode laser. Similarly an intracavity β-$B_aB_2O_4$ crystal [17] could serve to upconvert ultraviolet pulses from an independent intracavity frequency doubler by mixing with externally injected visible pulses, possibly the fundamental pulses from the same laser. These and other possibilities suggest that further study of intracavity nonlinear processes in passively mode-locked lasers may open up a wide new class of wavelength—extended femtosecond sources.

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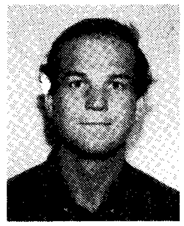

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